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ABSTRACT


In this paper we focus on the market-consistent valuation of embedded options within a unit-linked product. We will especially focus on unit-linked products with life cycle investment policies. For these contracts analytical valuation formulas are not available. We will explain, step-by-step, how one can value these embedded options accurately through the use of Monte Carlo simulation. Additional sensitivity analyses are presented to gain insight in the impact of variations in specific economic variables on the value of the embedded options.

Keywords: embedded options, life cycle investment policy, Monte Carlo valuation, unit-linked guarantee option

JEL Classification: G12; G13; G22

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1 Introduction

Most insurance contracts contain 'embedded' or 'implicit' options. An example of such an option is a guaranteed return or a guaranteed capital for a unit-linked investment policy. In this case, the insurance company promises to pay a certain minimum amount after the contract terminates. This way, the policy holder is shielded against negative economic scenarios, but will be able to profit from years of economic prosperity. Apart from guarantee options, there are several other types of embedded options in insurance contracts, for example contracts which offer the policy holder profit sharing in case the interest rate rises above a certain level. All in all these embedded options can be of significant value and therefore represent a liability for the insurance company.

One of the most striking examples of the potential impact of embedded options is the downfall of the insurance company Equitable Life in the United Kingdom (see O'Brien (2006)). When this company had to stop taking new business in 2000 this was largely due to the inadequate hedging of the written options. This incident has raised the awareness among insurance companies about the possible implications these options could have on the viability of the firm.

Therefore, in recent years the focus on embedded options has intensified. This increased attention is for instance reflected in recent regulatory processes in Europe like IFRS II and Solvency II. These new guidelines all emphasize the fair valuation of embedded insurance options. For example, the available capital may well decrease under Solvency II because the market value of 'embedded' options in insurance contracts (like profit sharing mechanisms or minimum return guarantees) needs to be considered explicitly. This stresses the need to properly account for the (potentially very high) costs of these embedded options when pricing insurance products. Apart from the regulating authorities rating agencies are also a factor in the increased focus on embedded options. These require ever more insight into the financial position of insurance companies. Given these different developments, insurance companies have become more focused on the correct pricing and risk management of products that contain embedded options.

In this paper the valuation of an embedded option within a unit-linked product is studied.³ A complicating characteristic of the product under consideration is its investment policy, which contains a life cycle element. This makes the use of analytical formulas as proposed by Schrager et al. (2004) unsuitable to value this particular type of embedded option. The methodology that is developed in this paper is more generally applicable and can also be used for a wide range of other options contained in insurance liabilities.

The remainder of this paper is organized as follows. In Section 2 we will briefly introduce the unit-linked product under consideration. Section 3 is concerned with the valuation framework which is used to determine the value of the embedded option and presents the results. In Section 4 several sensitivity analyses are conducted and Section 5 discusses how a break-even analysis can be used to determine an appropriate fee structure for the considered product. Section 6 concludes.

³ Potter (2003) is an early paper about this topic. Potter also stresses the importance of properly valuing and hedging guarantees in unit-linked policies.

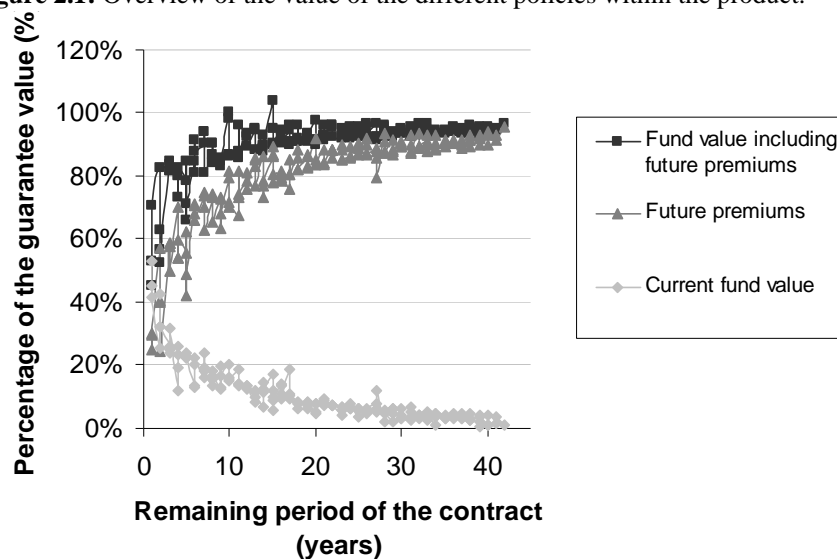
2 Product characteristics

A unit-linked life insurance product is a contract between a policy holder and an insurance company. The policy holder will contribute either a lump sum or a regular premium which will be invested by the insurance company. Furthermore, the insurance company can also promise a guaranteed amount or a minimum return on the invested premiums (after subtraction of fees) In other words, the insurance company promises to pay out a guaranteed amount at expiration of the contract. This guarantee can thus be seen as a (put) option written by the insurance company. If the fund value is below the guaranteed value at expiration, the option will be 'in-the-money' and the insurance company will have to settle the difference. Therefore the value of the guarantee is in essence a liability for the insurance company.

2.1 Contracts

The product under consideration in this paper has a 3% return guarantee on all invested premiums. The product is relatively new which means that the majority of the contracts still have a long time to maturity. In the following figure a breakdown of the different policies is presented.

Figure 2.1: Overview of the value of the different policies within the product.



The line at the bottom of the graph represents the current fund value (as a percentage of the guaranteed value at expiration) of the different policies. The maturity of the policies ranges from a few months to more than 40 years. The line in the middle indicates the total value of the premiums that still have to be paid for each specific policy (also as a percentage of the guaranteed value at expiration). Especially for the contracts which have a long time to maturity the future premiums account for the largest part of the guaranteed value. The line at the top of the graph combines both the current fund value and the future premiums. In an ideal world this value would be larger or equal to the value of the guarantee. However, as can be seen from the figure above, especially for policies with a short time to maturity the sum of the current fund value and future premiums is below 100%, indicating that these funds probably won't be sufficient to redeem the value of the guarantee in this example. In these cases the insurance company has to pay an additional amount if the situation does not improve before expiration.

2.2 Management costs

In order to cover the costs associated with this product, each policy is charged with a fixed management fee on an annual basis. This management fee equals 1.0% of the fund value for each policy and it is charged for two reasons:

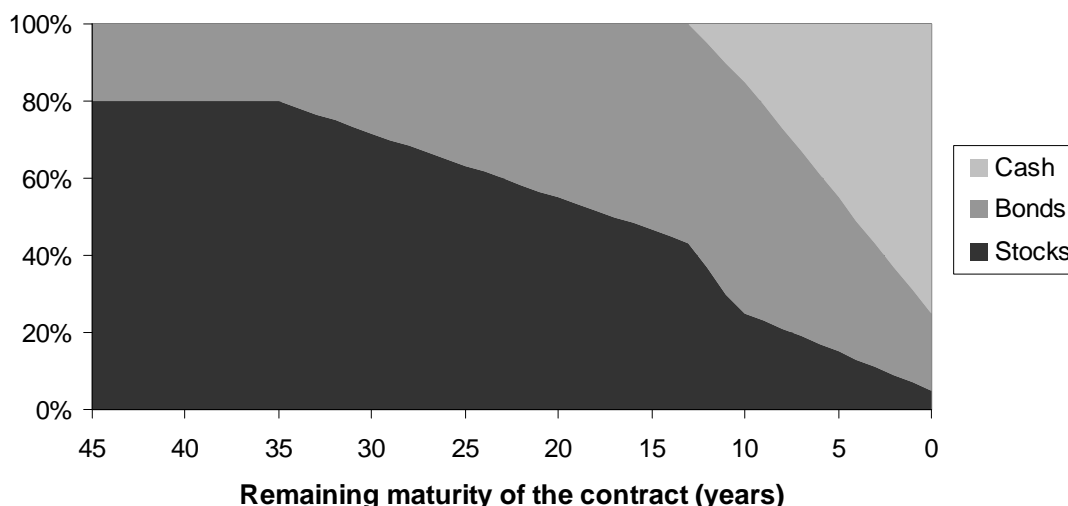
1. 85 base points are subtracted from the fund as an asset management fee.
2. The remaining 15 base points are subtracted to cover the value of the guarantee.

Hence, the management fee does not only account for the direct costs to the insurance company (e.g. the costs for managing the assets of the policy holder), but also for costs that might occur at expiration of the contract (e.g. when the fund value falls short of the value of the guarantee).

2.3 Investment choices and investment issues

Each policy holder invests either in stocks, bonds or cash; no other investment categories are available in this example. Furthermore, the policy holder is not able to choose freely between them since he has to take part in the life cycle that is defined for this product. The investments in stocks consist of US stocks (45%), Euro stocks (35%) and Japanese stocks (20%) and the investments in bonds are divided equally between Europe and the US.

Figure 2.2: Definition of the life cycle.



With respect to the valuation of the embedded option within this product there are two complicating factors.

1. The fact that the investment fund underlying the product incorporates a life cycle means that a different asset mix applies to almost every policy. This rules out the use of analytical formulas (see Schrager et al. (2004)) when determining the value of the embedded option. In order to obtain a correct market-consistent valuation Monte Carlo simulation has thus to be used (see Section 3).

2. The valuation is also complicated by the underlying asset allocation. The investment portfolio consists of categories denoted in several different currencies (e.g. Euro, US Dollar and Yen). Hence, both the exchange rates and interest rates corresponding to these currencies have to be taken into account.

3 Risk-neutral Monte Carlo valuation

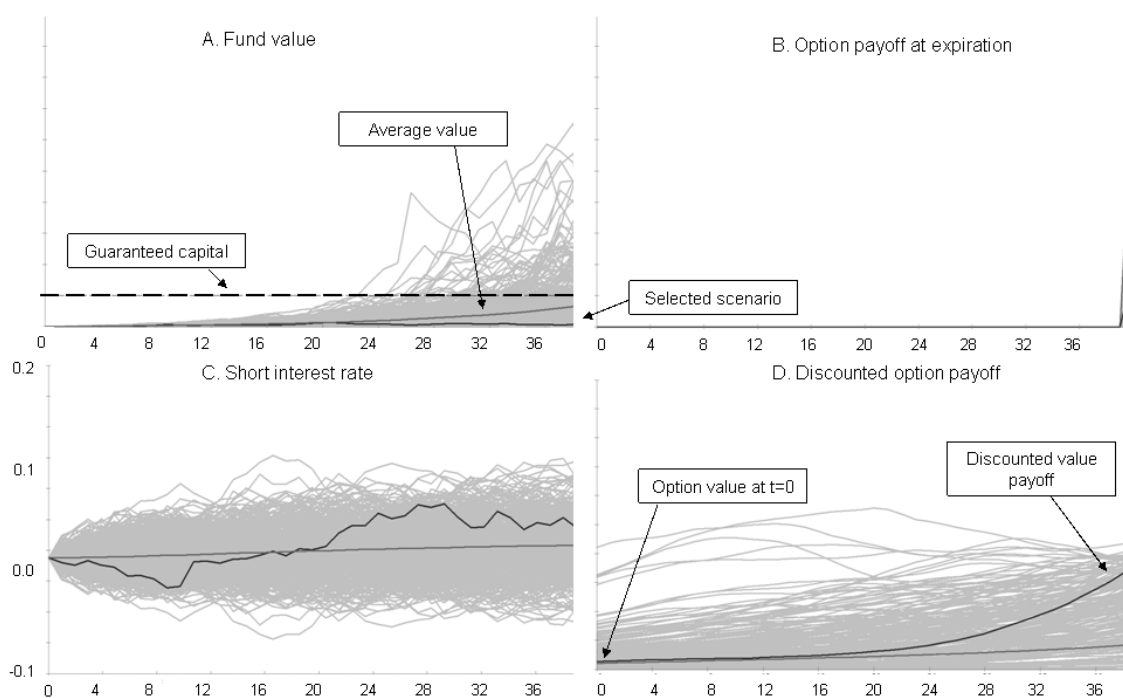
3.1 Risk neutral valuation

In a risk neutral world all individuals are indifferent to risk and expect to earn a return equal to the (instantaneous) risk-free rate on all securities. Assuming that the world is risk-neutral greatly facilitates the valuation of options: the option payoffs can simply be determined in a Monte Carlo experiment and discounted along the path of the short rate for each scenario. It is important to note that risk-neutral valuation gives the correct price of an option in *all* worlds (also the risk-averse world), not just in the risk-neutral world.

To correctly value the embedded option through Monte Carlo simulation a set of risk-neutral scenarios is needed. For each of these scenarios a specific policy can be simulated until expiration. By discounting the cash flows at expiration along the path of the short term interest rate, the value of the embedded option for this specific scenario is obtained. If the set of risk-neutral scenarios is sufficiently large a reliable value of the option can be obtained by averaging the different option values over all the scenarios under consideration. We will be using a set of 10,000 scenarios in the example presented here.

An example of the Monte Carlo valuation technique is given in the figure below. For simplicity we only consider a single policy which expires after 40 years. For the selected (dark grey) scenario the stock fund has reached a very low level after 40 years (see panel A). As a result, the value of the investments is lower than the guaranteed capital. This means that the guarantee option pays out for the policy holder (see panel B).

Figure 3.1: Example of the Monte Carlo valuation of a unit-linked guarantee



As stated before, this payoff is then discounted back along the path of the short interest rate. Instead of using a fixed interest rate for discounting the option payoffs, we use a stochastic interest rate (see panel C and D). The same calculation is performed for each scenario. The present value of the embedded option is then found by averaging all discounted option payoffs at the current point in time. The average discounted payoff is indicated by the black line.

The next section will discuss the model used to generate the required risk-neutral scenarios.

3.2 The Hull-White Black-Scholes model

The model that is used for generating risk-neutral scenarios is a combined Hull-White Black-Scholes model⁴. The Hull-White model is used to generate the interest rates whereas the Black-Scholes model is used to generate the asset returns. These two models can be combined since the expected asset returns are dependent on the short term interest rate in a risk-neutral world.

Since we want to perform a market-consistent valuation of the embedded option in the unit-linked product, the parameters of these models will be calibrated on actual market data such as interest rate curves and implied volatilities of swaptions. In case market information is not available (e.g. for stock options with long maturities) historical data will be used.

3.2.1 Calibration of the interest rate model

The first step in calibrating the interest rate model is the construction of the initial zero-coupon interest curve. In order to obtain this zero-coupon curve we will use (par) swap rates from Bloomberg with maturities ranging from 3 months to 30 years as the reference interest rate. For maturities longer than 30 years we assume the par interest rate curve to be flat. The par interest rate curve is converted into a zero-coupon curve by means of a bootstrapping procedure⁵. Our valuation date is 30/06/2009.

After having derived the zero-coupon interest rate curve, the stochastic (Hull-White) interest rate model can be calibrated. There are essentially two parameters that have to be calibrated for the one-factor Hull-White model which we use in this paper: a mean reversion parameter and a drift parameter. We also used a more elaborate two-factor Hull-White model. This does not improve the results significantly, however, so we continue with the simple one-factor model. In order to correctly calibrate the parameters on market data, forward at-the-money options on swap contracts (swaptions) will be used. More specifically, we will be using a large set of swaptions, with option and swap maturities ranging from 1 to 30 years. This calibration procedure will be repeated for each of the three currencies under consideration.

⁴ See Hull, 2009 (p. 277-309 and p. 688-690).

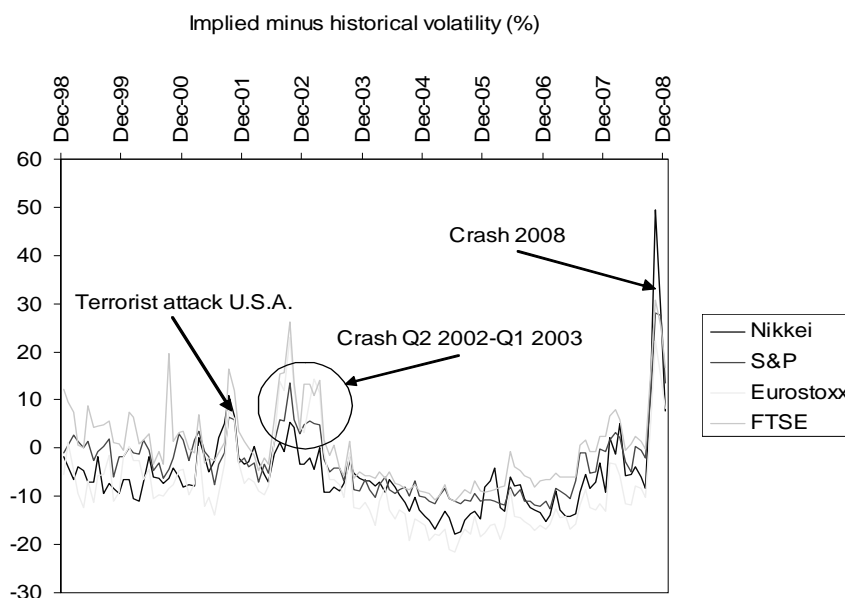
⁵ See Hull, 2009 (p. 82-84).

3.2.2 Calibration of the stock model

After the calibration of the interest rate model has been completed, the calibration of the stock model remains. The long-term volatility for the different stock indices will be based on the historical returns over a period of at least 20 years. As stated before, the historical volatility is used in this case since there are no stock options with very long maturities. In addition, market information from several option markets is used to adjust the short-term volatility for each of the stock indices under consideration. For example, options traded on the Eurostoxx are used to calibrate the short-term volatility in the Euro zone.

By comparing the implied volatilities of at-the-money put options with the historical volatility of the underlying indices, the difference between the actual (market-implied) volatility and the (long-term) volatility of the index can be evaluated. In the figure below this comparison is made for several major indices. Note the dramatic effect of stock market panic on the market-implied volatilities (compared to the long-term historical volatilities).

Figure 3.2: Difference (in percentage points) between the implied volatility of (short-term) at-the-money put options and the historical volatility of the underlying index.



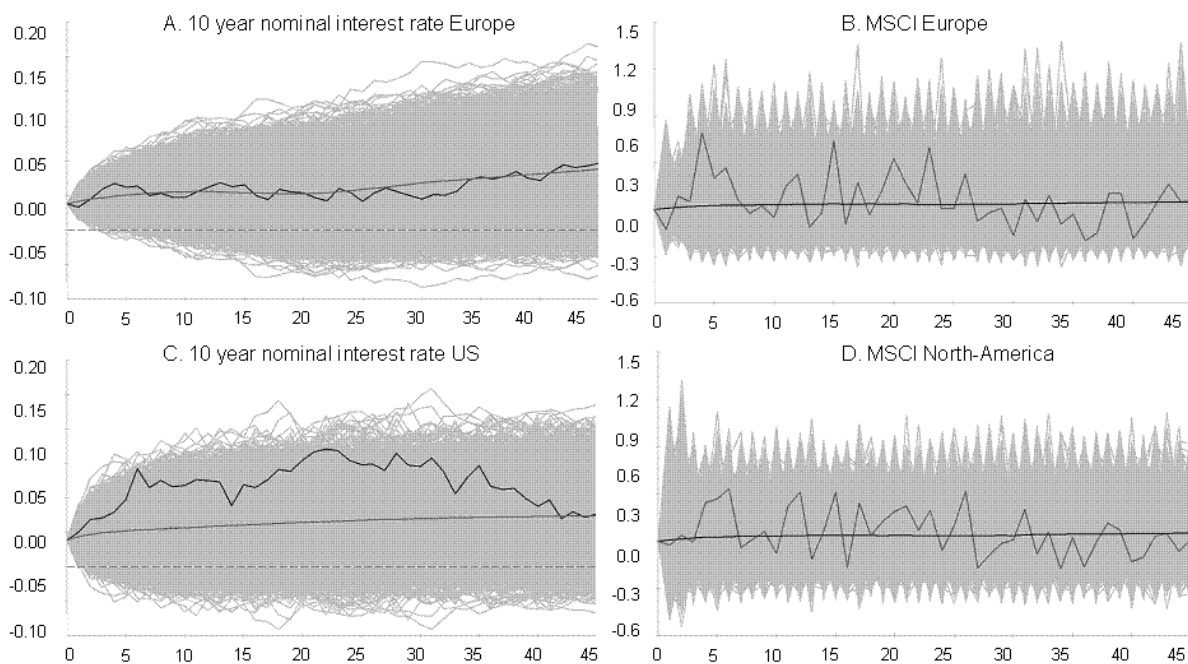
3.3 Scenario characteristics

When the interest rate model and the stock model have been calibrated, we are able to generate the 10,000 risk-neutral scenarios needed to value the embedded option. To get an impression of the results, the scenarios for a few selected variables are presented in the figure below. The scenarios are generated for a horizon of 45 years. Since there is no policy with a longer maturity, this horizon will be sufficient for our purpose.

Panel A and C show the evolution of the 10 year nominal interest rate, both in Europe and the US. Notice that the probability of negative interest rates is small given the interest rate curve and (market-

implied) interest rate volatilities used for calibrating the interest rate model. Note furthermore that the average interest rate is increasing over time. This effect is typical for risk-neutral valuation models and is commonly referred to as a 'convexity correction'⁶. Because of this correction the mean interest rate level increases if the volatility increases. Without this correction the prices of zero-coupon bonds with a long maturity do not match with the prices according to the initial interest rate curve. Therefore this correction is necessary for valuation purposes, since the no-arbitrage condition should be satisfied in this case. Panel B and D show the evolution of the corresponding MSCI indices. Both return series are expressed here in their local currencies. Furthermore these figures illustrate the use of market information from several option markets to adjust the short-term volatility for each of the stock indices. For example the volatility in the first two years of the MSCI North-America is significantly higher than the volatility in the following years.

Figure 3.3: Graphical representation of the generated risk-neutral scenarios.



3.4 Results

After having obtained the risk-neutral scenarios for the interest rates and stocks, we are now ready to calculate the results of the Monte Carlo simulation and thereby obtain the value of the embedded option for the unit-linked product under consideration.

⁶ See Hull (2009).

The result of the simulation can be found below.

Table 3.1: Results of the Monte Carlo simulation (*x* € 1.000).

| <i>Value embedded option</i> | <i>Standard deviation</i> | <i>St. dev. (% option value)</i> | <i>Guarantee provision</i> |
|------------------------------|---------------------------|----------------------------------|----------------------------|
| € 19,400 | € 90 | 0.5% | € 6,900 |

The value of the embedded option is equal to 18.7% of the total market value of the investments. So the embedded option actually is quite a sizeable portion of the total liability for this product. The value of the option can be largely traced back to the policies with a relative short time to maturity, all of which are expected to fall short of their guaranteed capital and for which the insurance company will have to bridge the gap. Furthermore note that the guarantee provision covers only about 35% of the total value of the embedded option. This indicates that the 15 base points, which are retained by the insurance company to actually cover the value of the guarantee, are insufficient.

The standard deviation of the option value is relatively small as can be seen in the above table. The standard deviation provides an indication of the reliability of the resulting option value. If the standard deviation (relative to the option value) is too large the number of scenarios used in the Monte Carlo simulation has to be increased. Given the results obtained in this example the 10,000 scenarios used to calculate the option value are sufficient to provide an accurate valuation (we have an error of 0.5%). Increasing the number of Monte Carlo scenarios will reduce the standard deviation even further if desirable.

Note that the market-consistent valuation of the embedded option is strongly dependent on the specific moment in time that one has chosen for performing the valuation. Due to changing market circumstances and changes in the liability portfolio over time it only provides a 'snapshot' value at one specific moment.

4 Sensitivity analyses

As stated before, the valuation of the embedded options is highly dependent on the specific moment in time for which the calculation is performed. The risk-neutral scenarios underlying the Monte Carlo simulation are calibrated on market data which changes over time. When the economic climate changes the value of the embedded option will therefore change as well.

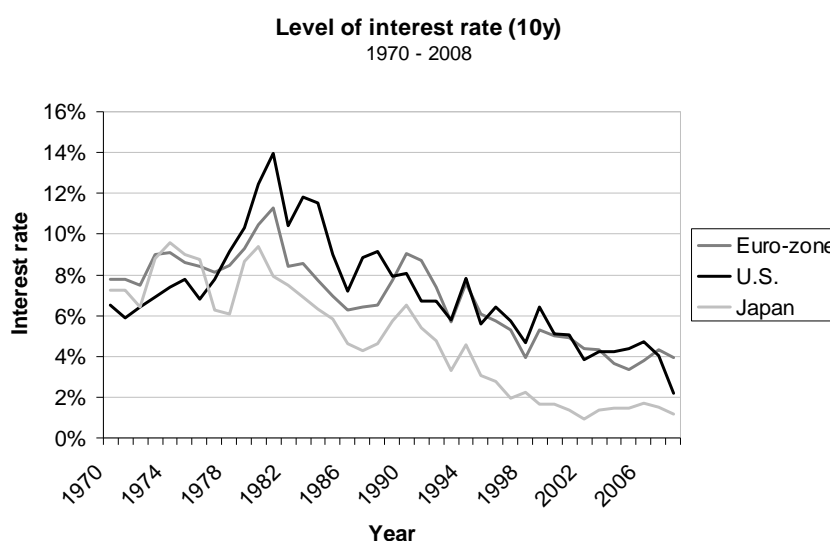
Hence, it is advisable not only to look at the current value of the embedded option, but also to analyze the effect on the value of the guarantee in case certain economic variables change. In these so-called sensitivity analyses one of the economic variables is varied, while all other variables remain unchanged (*ceteris paribus*). This way the impact of a specific variable is highlighted and one gains more insight into the dynamics behind the changes in the value of the embedded option over time.

4.1 Sensitivity with respect to the level of the interest rate

First we make a more detailed analysis of the interest rate sensitivity of the unit-linked product under consideration. Since we are considering an investment portfolio consisting of categories denoted in several different currencies (e.g. Euro, US Dollar and Yen) we consider a change in the level of all relevant interest rates.

Historically there has been a high correlation between the level of the interest rate for different currencies, as can be seen in the figure below.

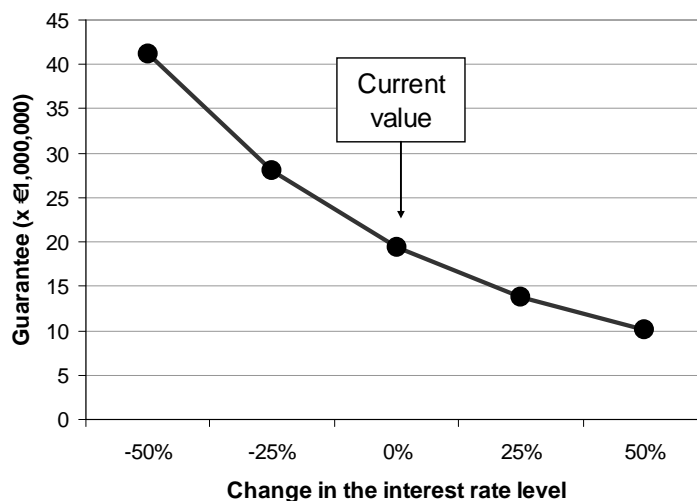
Figure 4.1: Interest rate levels across different currencies.



Given this high correlation, an identical shift is applied to all three interest rate levels (Euro, US and Japan) at the same time to gain insight into the sensitivity of the option value toward the level of the interest rate curve. To illustrate this a few alternative economic scenario sets are created. These alternative scenarios are essentially identical to the initial set with the exception of the level of the

interest rate. Compared to the original economic set this level is altered by -500 base points to +500 base points. The results can be found in the figure below.

Figure 4.2: Impact of a change in the interest rate level.



The current value of the guarantee corresponds to the point lying at '0%' on the horizontal axis. To the left and right are the values of the guarantee in case of respectively a decline or rise in the level of the interest rate. This figure illustrates clearly that the value of the guarantee is very sensitive to a change in the level of the interest rate.

The value of the guarantee especially increases at low interest rates. The reason for this sharp increase in the value of the embedded option is twofold. First, due to the lower interest rate level future capital shortages will be discounted with a lower yield. This will result in a higher present value of all future capital shortages. Second, the return on stocks in a risk-neutral world will be lower in case of a lower interest rate. When the return on stocks decreases the probability of a future capital shortage will increase. This will further increase the value of the embedded option. The reverse reasoning holds for the option values at higher interest rates.

4.2 Sensitivity with respect to the implied volatility of stocks

Besides the level of the interest rate, the value of the guarantee is also dependent on the annual investment return. For instance, if a policy is close to its expiration date a negative investment return could cause the fund value to fall below the guaranteed capital. Since the policy is close to expiration there is little time to recuperate from this setback, causing the value of the guarantee to rise.

In order to illustrate the effect of the sensitivity toward the volatility of the investment returns, we again consider several alternative scenario sets. In comparison to the original scenario set we change the volatility of the stock returns, again keeping all else equal. The results of this analysis can be found in Figure 4.3.

Again, the value that corresponds to '0%' on the horizontal axis represents the current value of the guarantee. Increasing the volatility of the stock returns with an additional 5% (in absolute terms) leads to an increase in the value of the guarantee. This is because an increase in the volatility makes the overall investments more risky, which is reflected in the value of the guarantee.

Note, however, that the sensitivity toward the volatility of the stock returns is much less than the sensitivity toward the interest rate level. This is due to the nature of the product. Since the policies contained within this product on average have a long time to maturity this means they are less susceptible to a single negative annual return: there are still plenty of years remaining to recuperate from a 'bad' year. For portfolios with shorter maturities the value of the guarantee becomes more dependent on the volatility of the stock returns.

Moreover, the sensitivity toward negative investment returns is diminished further because all investments are made with respect to the predefined life cycle. This means that all policies which are close to expiration invest according to a less risky allocation (e.g. almost no stocks). Therefore the possibility of a strongly negative return is reduced which in turn reduces the value of the guarantee. This can be seen in Figure 4.4.

The investment portfolio underlying the unit-linked product is set equal to an average investment mix in case a life cycle is not used. This average mix is derived from the life cycle and equals the investment mix at the average time to maturity of all separate policies.

Figure 4.3: Impact of a change in the volatility of stock returns.

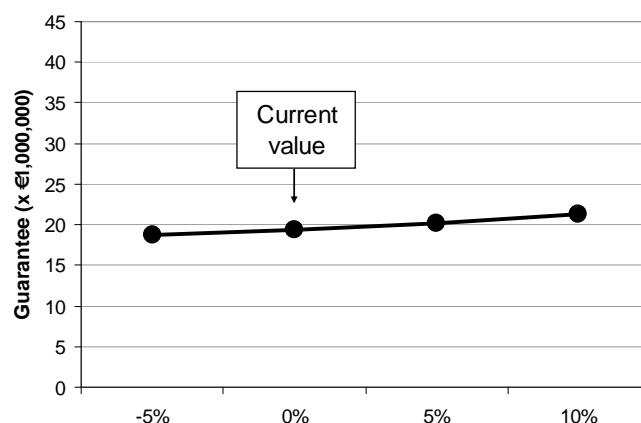
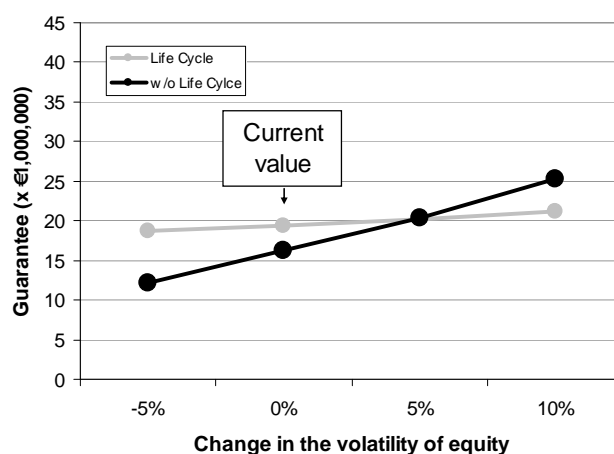


Figure 4.4: Impact of a change in the volatility of stock returns.

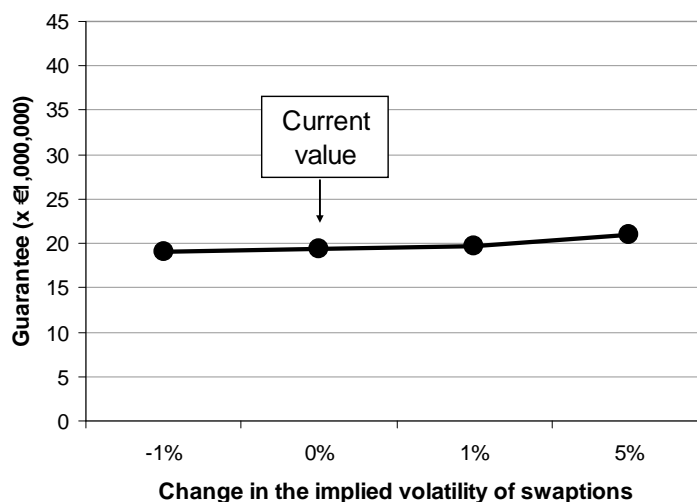


4.3 Sensitivity with respect to the interest rate volatility

Finally we consider the sensitivity of the value of the guarantee toward the volatility of the interest rate. If the volatility of the interest rate is relatively high, there is an increased probability of the interest rate reaching lower levels. Because of a relatively low degree of mean reversion in the calibrated interest rate scenarios the interest rate will then remain at this lower level for a certain period of time with a large probability. This will cause an increase in the value of the embedded option.

In order to gain insight into the sensitivity toward the volatility of the interest rate, we once again consider several alternative scenario sets. Keeping all else equal, these scenario sets are constructed by increasing and decreasing the (market-based) implied volatilities of swaptions which are used to calibrate the Hull-White model. Note that we consider an identical increase/decrease in the implied volatilities of swaptions for each currency.

Figure 4.5: Impact of a change in the implied volatility of swaptions.



The impact of a change in the implied volatility of the swaptions used to calibrate the Hull-White model is modest and smaller than a change in the level of the interest rate or the volatility of the stock returns.

5 Break-even analysis

Based on the simulation and valuation techniques presented before, an insurance company is able to gain insight into the (expected) costs associated with this specific unit-linked return guarantee. Given this knowledge, the insurance company will be able to determine the appropriate fees to charge for the return guarantee under consideration, taking into account the risk and return characteristics of the life cycle, the age of the policy holder and the maturity of the contract.

Using a “break-even analysis” the value of the embedded option can be stated in terms of the number of base points that should be subtracted from the realized return of the investment fund. For this subtracted number of base points it holds that the option value of the guarantee equals the present value of the withdrawals. Based on these results an insurance company will be able to make more informed decisions while designing the product, regardless of the complexity of the proposed investment strategy.

To illustrate this analysis, we consider two different investment strategies. The first investment strategy is equal to the life cycle as presented before. The second strategy is a mixed fund that is based on the life cycle and is equal to an average investment mix. Furthermore we consider a young (20 year old) policy holder and an older (40 year old) policy holder. We assume that in both cases the contract will expire at the age of 60.

The following table represents the value of the guarantee in terms of the number of base points that should be withdrawn from the realized return of the fund.

Table 5.1: Results of the break-even analysis for several cases.

| | | | Guarantee | |
|-----|----------|------------|-----------|-----|
| Age | Maturity | Fund | 0% | 3% |
| 20 | 40 | Mixed | 9 | 101 |
| 20 | 40 | Life Cycle | 4 | 78 |
| 40 | 20 | Mixed | 7 | 113 |
| 40 | 20 | Life Cycle | 4 | 92 |

Notice that the life cycle investment policy, as expected, leads to the best results.

6 Conclusion

Calculating the fair value of options which are embedded in insurance contracts is necessary for several applications. For instance, under Solvency II and IFRS II the fair value of the options needs to be considered explicitly. But also for periodic financial reports, economic capital calculations as well as ALM analysis, a correct valuation of these options is crucial.

In this paper we demonstrate how complex embedded options within unit-linked insurance contracts can be valued. As an example we have looked at a unit-linked product whose investment policy contains a life cycle element. Because of this life cycle element analytical formulas cannot be used to obtain a market consistent valuation of the embedded option. For this reason we need to use Monte Carlo simulation. Note that this methodology can also be used to value a wide range of other options contained in insurance contracts (like profit sharing options).

We first generate the risk-neutral scenarios needed to perform the Monte Carlo simulation. In order to generate these scenarios we calibrate a Hull-White Black-Scholes model on market data. For the calibration of the Hull-White model swaption prices are used, whereas the Black-Scholes model is calibrated on the historical volatility since no stock options with very long maturities exist. Additionally, market information from option markets is used to adjust the short term volatility. Given these scenarios the value of the embedded option is calculated by averaging all discounted option payoffs at the current point in time.

Besides determining the value of the embedded option at a specific moment in time, additional analyses provide insight into the sensitivity of the option value with respect to certain economic variables. By varying one of these variables, while keeping all others unchanged (*ceteris paribus*), the impact of the specific variable is highlighted and one gains insight into the dynamics behind the changes in the value of the embedded option over time.

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